STUDENT SEMINARS ON "FAMOUS EQUATIONS"

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> Geometry has two great treasures: one is the theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel.
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> — Johannes Kepler [1571–1630]

Treasures often lie obscure in mathematics programs constrained by rigid syllabi and taxing workloads. This report describes a practical way for students and faculty to examine some of the golden threads and sparkling jewels which are woven into our mathematical heritage.

Each winter term a small troop digs through the library stacks for information about famous equations. These diggers are participants in our Famous Equations Seminar-course. Each is looking for the general lore and specific features of a personally chosen equation, the makings of a seminar talk and term paper.

This course is overtly intended to counter the impression that mathematics somehow sprang full-grown into the bindings of textbooks; and that the only way to learn math is to study the text and listen to an instructor.

Credit participants meet once each week for two hours. The first session emphasizes the collegium nature of a seminar, puts in a plug for history, and discusses the broad meaning of "an equation." During the next few sessions, faculty talks are presented on a famous equation or two, providing reasonable models for the students as they prepare their own presentations. The student talks follow, one each week. All talks are open to the public. The final session recaps and gathers common threads from the series' talks, and returns to the philosophic issues raised in the opening session. It is also the time for students to exchange final versions of the papers they have written; each student departing with an anthology of "famous equations."

The theme of "famous equations" has proven ideal; it is catchy, permits well-focused individual investigations, and is nicely comprehensive. As our hallway poster expounds, "Over the centuries certain 'truths' have been discovered and (almost too) neatly packaged into now famous equations such as:

$$x^n + y^n = z^n \qquad \int_0^\infty e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2}$$

$$\overline{Q} = R \qquad V - E + F = 2$$

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}} \qquad A = \pi r^2$$

$$c = 2^{\aleph_0} \qquad C = R[\sqrt{-1}]$$

and $e^{i\pi} + 1 = 0$. The ideas captured by these germinal equations are body and soul for much of mathematics. The struggle to formulate these equations is a tribute to human persistence in the search for understanding." A student tracking down the heritage of any one of these equations is quickly exposed to the humanistic side of mathematics evolution.

Novel to many students is the idea that they are central in the information gathering and sharing process. A letter to prospective participants sets the cooperative tone of the series, and the course "Guidelines for Papers and Talks" pointedly reminds them that their audience is themselves. The Guidelines also indicate the appropriate mix of lore and rigor, and the expected level of performance. (A copy of the Guidelines is appended.)

The two-hour meetings allow for an hour talk, a break, and an open discussion about the newly presented information and earlier talks. The post-talk discussions are extremely valuable; into them often pop bits of information not included in the prepared talks. And it is especially during these informal exchanges that participants come to know and respect one another.

In conclusion, the seminar-course format recommends itself for several reasons:

(1) It is practical. Participants readily fit the series into their schedules, and are able to do much of the required work well in advance. The instructor needs to select participants, compile a partial list of possible equations with a few initial references, give a couple of talks, and carefully critique eight 15–20 page papers.

(2) It is adaptable. The level of equations may range from $1+1=2$ to $\beta X = X$.

(3) It is immersive. The students together discover cross-threads of history. ("What, Euler again!" or "Your Fibonacci is in my Pascal triangle!")

(4) It is self-regulatory. Participants have automatically striven to do quality papers and talks for the benefit of their peers.

(5) It is naturally fortifying. It provides genuine (albeit fledgling) experiences in library research and in professional writing and speaking.

(6) It is fun!

Most importantly, the format fosters learning without a textbook and brings people together for the purpose of sharing information about a common interest — surely treasures of humanistic learning.

APPENDIX — Famous Equations
GUIDELINES FOR PAPERS AND TALKS
MATH 399 HONORS: FAMOUS EQUATIONS
WINTER 1987

PAPER

First draft due February 23.
Final paper due March 9.

This is to be an exposition written for your fellow students. As you write you should imagine the typical reader to be an upper-division mathematics student with some ability and curiosity. Your job is to organize, summarize, and verbalize what you find out about your equation.

The paper should contain both general lore and specific features. By "general lore" we mean such things as historical appearances, significance at that time and later, personages involved, and interesting uses. By "specific features" we mean the statement and details of interesting theorems, mathematical arguments, unexpected applications, variations or generalizations. On proofs, use your judgement about how much detail should be included in the text itself. Sometimes the details are the heart of understanding; other times they should be relegated to an appendix.

Be sure your paper includes those things which, when first encountered by you, evoked reactions such as: "That's neat!," "Clever!," "I don't believe it!," "Hmm..." or "Curious!"

It is expected that an Honor's paper will conform to proper English practices. The paper should contain an introduction and a conclusion. References tagged to the text are essential and a briefly annotated bibliography is very desirable.

The use of ugly sentence construction and sloppy punctuation are cause for flogging.

You do not need to be flowery with words. It is important to be direct, uncluttered and clear.

The paper should be 15–20 pages (not counting appendices and bibliography).

TALK

Your talk is another way to tell your friends about your equation and its curiosities. It should not be treated as a quick reading of your paper; it is an entirely different medium.

Spend some time telling us (the audience) about your equation and its "general lore." Then take one "specific feature" which you found particularly interesting and sketch details with enough comment and discussion to illuminate its significance and finer points.

Prepare well ahead of time! Think about what we (the audience) don't know, then plan a progression to enlighten us. Once you start your talk, treat it as a casual, but directed, chat with your friends.